EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION

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Keywords: Evolutionary optimization, Multi-objective optimization, Multi-criterion decision making, Genetic algorithms, Innovation.

Contents

Summary

In the past three decades, evolutionary algorithms (EAs) have been found to be extremely useful in solving various search and optimization problems. Although much of the early advancements and applications concentrated in solving single-objective optimization problems, researchers realized the potential and niche of EAs is handling multi-objective optimization problems vis-a-vis their classical counterparts. Suggested in the beginning of nineties, evolutionary multi-objective optimization (EMO) algorithms are now routinely used in solving problems with multiple conflicting objectives in various branches of engineering, science and commerce. In this chapter, we provide an overview of EMO methodologies by first presenting principles of EMO through an illustration of one specific algorithm and its application to an interesting real-world bi-objective optimization problem. Thereafter, we provide a list of recent research and application developments of EMO to provide a picture of some salient advancements in EMO research. The development and application of EMO to multi-objective optimization problems and their continued extensions to solve other related problems has elevated the EMO research to a level which may now undoubtedly be termed as an active field of research with a wide range of theoretical and practical research and application opportunities. Hopefully, this chapter should motivate readers to pay more attention to their growing field of evolutionary multi-objective optimization methods and their scopes in practice.

1. Introduction

An optimization task is a computing process in which an intelligent search is performed in an usually large-dimensional space involving a number of *decision variables* (Boolean, discrete, real, permutations etc.) for locating a special point that would minimize or maximize a pre-specified *objective* which is a function of decision variables. The search space in most real-world problems is restricted by a number of *constraints* that are also functions of decision variables. The optimal point lies in the restricted search space, commonly known as the *feasible search space*. When objective and constraint functions are linear or convex functions of decision variables, provably fast optimization algorithms are available for locating the optimum point even for a large dimensional space. However, for arbitrary structures of objective and constraint functions, no single optimization algorithm can be equally efficient for all problems (Wolpert and Macready, 1997) , hence practitioners are better off in looking for an efficient algorithm for the problem at hand.

Optimization methodologies are useful in solving various types of practical problems. Some of them are presented below:

1. Optimal *design* problems in which the shape, connectivity, dimensions, materials etc. of the component or the system at hand are decision variables. The objective can be any design criterion, most common being the minimization of weight of the product or maximizing the life of the product or achieving some other functional goals. Constraints are usually involved with feasibility and safety of the product in terms of stress being less than or equal to strength, natural frequency being higher than forcing frequency. A lion's share of optimization efforts is spent in solving optimal design problems.
2. Optimal *manufacturing* process design problems in which process parameters are decision variables and the objective is often to minimize the overall processing time or maximize the surface finish or quality of the fabricated product. Constraints are often related to meeting available resources, due date of delivery, etc.
3. Optimal *control* problems for which variations of a few control parameters over time are decision variables and objective is often to minimize overall energy requirement, maximize the quality of output product, or minimize overall control time. Constraints involve in meeting specified by-products or meeting a specified value of one or more objectives mentioned above.
4. *Inverse* problems such as reconstruction or tomography problems for which a construction plan of available information (images or other data) becomes decision variable and the error between reconstructed structure and actual structure becomes an objective that is usually minimized. A physically viable and most simplistic reconstruction structure (known as Occam's razor (Soklakov, 2002) becomes constraints.
5. Data-driven *modeling* problems in which modeling structure and associated parameters become decision variables. The error in performances between model and the real object (or desired object) becomes an objective that needs to be minimized. Instead of allowing any arbitrary model to appear during the optimization process, some constraints relating the feasibility of components of the structure can be kept as constraints.
6. *Data-mining* problems in which classification, clustering, prediction, and forecasting related activities can also be solved using by posing them as a suitable optimization problems.
7. *Machine learning* tasks, in which one of the main activities is to develop intelligent

and self-adaptive systems, are often solved by posing the problems as optimization problems. Since optimal solutions are *special* points in the entire search space of possible solutions, optimization algorithms are intelligent procedures for arriving at these special solutions. Thus, it is not surprising that optimization algorithms can assist in finding an optimal configuration or a system that is self-adaptive and intelligent enough to arrive at human-competitive solutions.

Most such practical search and optimization problems usually involve nonlinear, non-convex and non-differentiable objective and constraint functions. They provide a stiff challenge to mathematically-motivated optimization algorithms even today. In such cases, the use of meta-heuristic optimization methods such as evolutionary algorithms (Goldberg, 2989; Holland, 1975; De Jong 2006), simulated annealing (Laarhoven amd Aarts, 1987), tabu search (Glover, 1989; 1990), and other methods motivated by another natural or physical phenomenon have been found to be useful. In this chapter, we describe multi-objective optimization algorithms based on the EA methodology.

EAs were traditionally used for solving problems having a single goal or objective. However, as evident from the above list of optimization problems, most real-world problems ideally involve multiple conflicting objectives, such as simultaneously minimizing cost of fabricating the product and maximizing its quality. Theoretically such multi-objective optimization problems give rise to a set of trade-off optimal solutions, known as *Pareto-optimal* solutions. Since classical optimization algorithms work with a single point in each iteration and deliver a single solution at the end of the optimization task, they need to be applied multiple times in order to find multiple Pareto-optimal solutions. This makes the application of classical optimization algorithms inconvenient for solving multi-objective optimization problems. On the other hand, EA's population approach makes them ideal candidates for solving multi-objective optimization problems.

The exploitation of EA's population approach in finding and maintaining multiple Pareto-optimal solutions was demonstrated during 1993-95 by three independent groups of researchers from Europe (Fonseca and Fleming, 1993), India (Srinivas and Deb, 1995), and USA (Horn et al., 1994). All three algorithms originated from David E. Goldberg's description of a 10-like sketch of a probable EA for multi-objective optimization (Goldberg, 1989). These studies were so exemplary and convincing that they in some sense gave birth to a new and promising field of computation: Evolutionary Multi-Objective Optimization (EMO). Subsequent to the three studies, EMO methodologies were made better, faster and more accessible. The algorithms were commercialized by various software companies and have made the field of EMO more popular and applicable to many different problems that academic researchers probably would not have achieved alone.

In this chapter, we provide a brief overview of the EMO principle, present one EMO algorithm in detail, and emphasize the importance of using EMO in practice. Besides this specific algorithm, there exist a number of other equally efficient EMO algorithms which we do not describe here for brevity. Instead, in this chapter, we discuss a number of recent advancements of EMO research and application which are driving the researchers and practitioners ahead. Fortunately, researchers have utilized the EMO's principle of solving multi-objective optimization problems in handling various other problem-solving tasks. The diversity of EMO's research is bringing researchers and practitioners together with different backgrounds including computer scientists, mathematicians, economists, engineers. The topics we discuss here amply demonstrate why and how EMO researchers from different backgrounds must and should collaborate in solving complex problem-solving tasks which have become the need of the hour in most branches of science, engineering, and commerce today.

2. Evolutionary Multi-objective Optimization (EMO)

A multi-objective optimization problem involves a number of objective functions which are to be either minimized or maximized subject to a number of constraints and variable bounds:

$$\left.\begin{array}{lll} \textit{Minimize / Maximize} & f_m(\mathbf{x}), & m = 1,2,\ldots,M; \\ \textit{Subject to} & g_j(\mathbf{x}) \geq 0 & j = 1,2,\ldots,J; \\ & h_k(\mathbf{x}) = 0 & k = 1,2,\ldots,K; \\ & x_i^{(L)} \leq x_i \leq x_i^{(U)} & i = 1,2,\ldots,n. \end{array}\right\} \quad (1)$$

A solution $\mathbf{x} \in \mathbf{R}^n$ is a vector of n decision variables: $\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$. The solutions satisfying the constraints and variable bounds constitute a *feasible set* S in the decision variable space $\mathbf{R}^n$. One of the striking differences between single-objective and multi-objective optimization is that in multi-objective optimization the objective function vectors belong to a multi-dimensional objective space $\mathbf{R}^M$. The objective function vectors constitute a feasible set Z in the objective space. For each solution $\mathbf{x} \in S$, there exists a point $\mathbf{z} \in Z$, denoted by $f(\mathbf{x}) = \mathbf{z} = (z_1, z_2, \ldots, z_M)^{\mathrm{T}}$. To make the descriptions clear, we refer a decision variable vector as a solution and the corresponding objective vector as a point.

The optimal solutions in multi-objective optimization can be defined from a mathematical concept of *partial ordering* (Schroder, 2003). In the parlance of multi-objective optimization, the term *domination* is used for this purpose. In this section, we restrict ourselves to discuss unconstrained (without any equality, inequality or bound constraints) optimization problems. The domination between two solutions is defined as follows (Deb, 2001; Miettinen, 1999):

Definition 1. A solution $\mathbf{x}^{(1)}$ is said to dominate the another solution $\mathbf{x}^{(2)}$, if both the following conditions are true:

8. The solution $\mathbf{x}^{(1)}$ is no worse than $\mathbf{x}^{(2)}$ in all objectives. Thus, the solutions are compared based on their objective function values (or location of the corresponding points ($\mathbf{z}^{(1)}$ and $\mathbf{z}^{(2)}$) in the objective function set Z).
9. The solution $\mathbf{x}^{(1)}$ is strictly better than $\mathbf{x}^{(2)}$ in at least one objective.

For a given set of solutions (or corresponding points in the objective function set Z, for example, those shown in Figure 1(a)), a pair-wise comparison can be made using the above definition and whether one point dominates another point can be established.

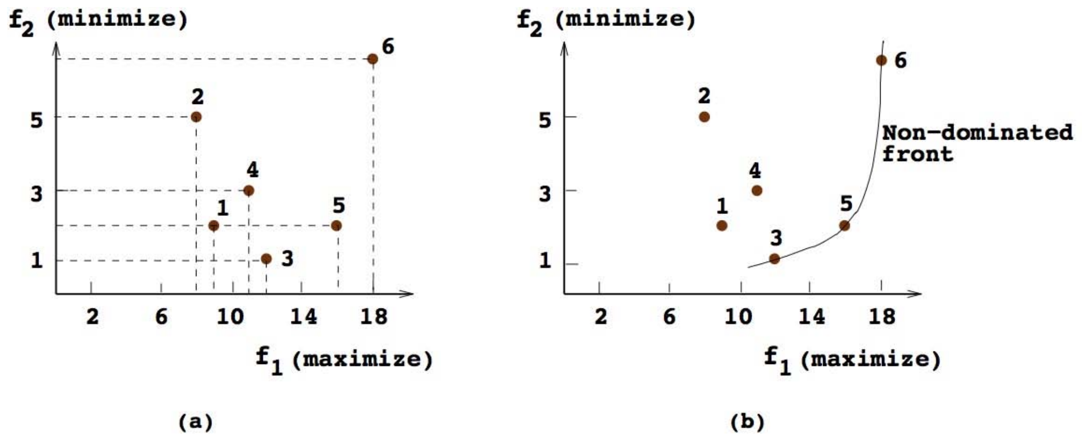

Figure 1. A set of points and the first non-dominated front are shown.

All points which are not dominated by any other member of the set are called the non-dominated points of class one, or simply the non-dominated points. For the set of six points shown in the figure, they are points 3, 5, and 6. One property of any two such points is that a gain in an objective from one point to the other happens only due to a sacrifice in at least one other objective. This *trade-off* property between the non-dominated points makes the practitioners interested in finding a wide variety of them before making a final choice. These points make up a front when viewed together on the objective space; hence the non-dominated points are often visualized to represent a *non-dominated front*. The theoretical computational effort needed to select the points of the non-dominated front from a set of N points is $O\left(N \log N\right)$ for 2 and 3 objectives, and $O\left(N \log^{M-2} N\right)$ for $M > 3$ objectives (Kung et al., 1975), but for a moderate number of objectives, the procedure need not be particularly computationally effecient in practice.

With the above concept, now it is easier to define the *Pareto-optimal solutions* in a multi-objective optimization problem. If the given set of points for the above task contain *all* feasible points in the objective space, the points lying on the first non-domination front, by definition, do not get dominated by any other point in the objective space, hence are Pareto-optimal points (together they constitute the Pareto-optimal front) and the corresponding pre-images (decision variable vectors) are called Pareto-optimal solutions. However, more mathematically elegant definitions of Pareto-optimality (including the ones for continuous search space problems) exist in the multi-objective optimization literature (Jahn, 2004; Miettinen, 1999). Interested readers are encouraged to refer to these references.

Obviously, the above definition and procedure of arriving at Pareto-optimal solutions is not a practical approach, as it involves finding all solutions in the search space. According to no-free-lunch theorem (Wolpert and Macready, 1997), since no single mathematical or classical optimization algorithm exists that would solve all single-objective optimization problems efficiently, the no-free-lunch theorem can also be extended for multi-objective optimization and a similar conclusion can be made (Corne and Knowles, 2000). Therefore, in solving arbitrary multi-objective optimization problems, our goal is use an efficient algorithm that would reach close to the true Pareto-optimal solutions. In Section 4 we present an optimization algorithm that in most problems consider only a tiny fraction search space and proceed near the Pareto-optimal solutions with iterations.

2.1. EMO Principles

In the context of multi-objective optimization, the extremist principle of finding the optimum solution cannot be applied to one objective alone, when the rest of the objectives are also important. This clearly suggests two ideal goals of multi-objective optimization:

- **Convergence:** Find a (finite) set of solutions which lie on the Pareto-optimal front, and
- **Diversity:** Find a set of solutions which are diverse enough to represent the entire range of the Pareto-optimal front.

EMO algorithms attempt to follow both the above principles, similar to a posteriori MCDM method. Figure 2 shows schematically the principles followed in an EMO procedure.

Since EMO procedures are heuristic based, they may not guarantee finding exact Pareto-optimal points, as a theoretically provable optimization method would do for tractable (for example, linear or convex) problems. But EMO procedures have essential operators to constantly improve the evolving non-dominated points (from the point of view of convergence and diversity mentioned above) similar to the way most natural and artificial evolving systems continuously improve their solutions. To this effect, a recent study (Deb et al., 2007) has demonstrated that a particular EMO procedure, starting from random non-optimal solutions, can progress towards theoretical Karush-Kuhn-Tucker (KKT) points with iterations in real-valued multi-objective optimization problems. The main difference and advantage of using an EMO compared to a posteriori MCDM procedures is that multiple trade-off solutions can be found in a single run of an EMO algorithm, whereas most a posteriori MCDM methodologies would require multiple independent runs.

In Step 1 of the EMO-based multi-objective optimization and decision-making procedure (the task shown vertically downwards in Figure 2), multiple trade-off, non-dominated points are found. Thereafter, in Step 2 (the task shown horizontally, towards the right), higher-level information is used to choose one of the obtained trade-off points.

All points which are not dominated by any other member of the set are called the non-dominated points of class one, or simply the non-dominated points. For the set of six points shown in the figure, they are points 3, 5, and 6. One property of any two such points is that a gain in an objective from one point to the other happens only due to a sacrifice in at least one other objective. This *trade-off* property between the non-dominated points makes the practitioners interested in finding a wide variety of them before making a final choice. These points make up a front when viewed together on the objective space; hence the non-dominated points are often visualized to represent a *non-dominated front*. The theoretical computational effort needed to select the points of the non-dominated front from a set of N points is $O(N \log N)$ for 2 and 3 objectives, and $O(N \log^{M-2} N)$ for $M > 3$ objectives [94], but for a moderate number of objectives, the procedure need not be particularly computationally effecient in practice.

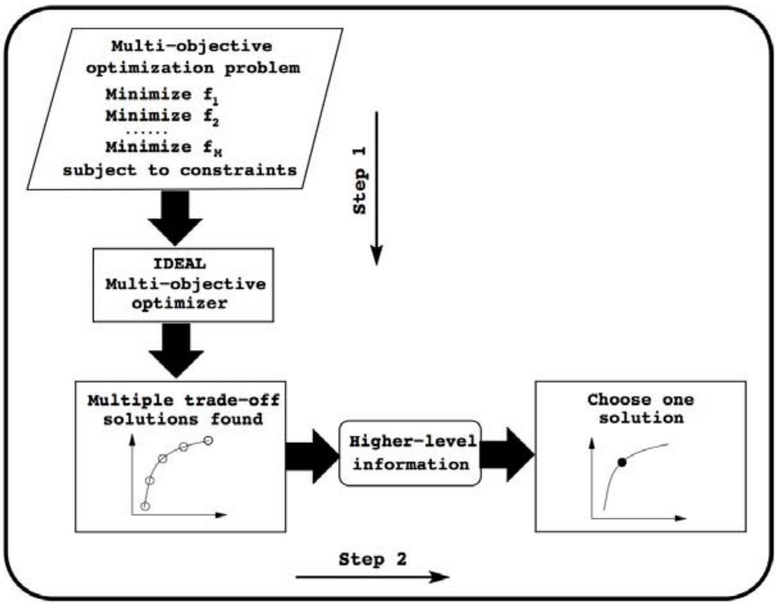

Figure 2. Schematic of a two-step multi-criteria optimization and decision-making procedure.

With the above concept, now it is easier to define the *Pareto-optimal solutions* in a multi-objective optimization problem. If the given set of points for the above task contain *all* feasible points in the objective space, the points lying on the first non-domination front, by definition, do not get dominated by any other point in the objective space, hence are Pareto-optimal points (together they constitute the Pareto-optimal front) and the corresponding pre-images (decision variable vectors) are called Pareto-optimal solutions. However, more mathematically elegant definitions of Pareto-optimality (including the ones for continuous search space problems) exist in the multi-objective optimization literature [103, 82]. Interested readers are encouraged to refer to these references.

Obviously, the above definition and procedure of arriving at Pareto-optimal solutions is not a practical approach, as it involves finding all solutions in the search space. According to no-free-lunch theorem [128], since no single mathematical or classical optimization algorithm exists that would solve all single-objective optimization problems efficiently, the no-free-lunch theorem can also be extended for multi-objective optimization and a similar conclusion can be made [27]. Therefore, in solving arbitrary multi-objective optimization problems, our goal is use an efficient algorithm that would reach close to the true Pareto-optimal solutions. In Section 4 we present an optimization algorithm that in most problems consider only a tiny fraction search space and proceed near the Pareto-optimal solutions with iterations.

2.2. A Posteriori MCDM Methods and EMO

In the 'a posteriori' MCDM approaches (also known as 'generating MCDM methods'), the task of finding multiple Pareto-optimal solutions is achieved by executing multiple independent single-objective optimizations, each time finding a single Pareto-optimal solution (Miettinen, 1999). A parametric scalarizing approach (such as the weighted-sum approach, ϵ-constraint approach, and others) can be used to convert multiple objectives into a parametric single-objective objective function. By simply varying the parameters (weight vector or ϵ-vector) and optimizing the scalarized function, different Pareto-optimal solutions can be found. In contrast, in an EMO, multiple Pareto-optimal solutions are attempted to be found in a single run of the algorithm by emphasizing multiple non-dominated and isolated solutions in each iteration of the algorithm and without the use of any scalarization of objectives. However, several EMO efforts have been put in finding multiple Pareto-optimal solutions in a single run using one of the above scalarizing methods (Jin et al., 2001; Hajela and Lin, 1992).

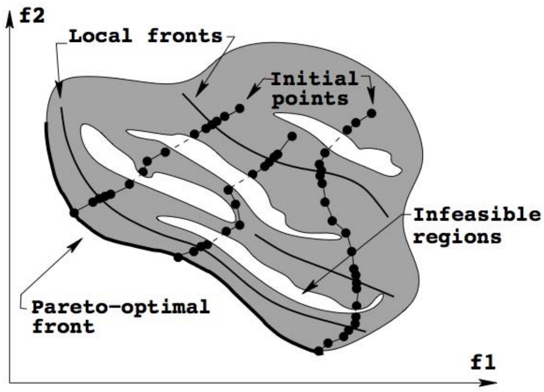

Figure 3: A posteriori MCDM methodology employing independent single-objective optimizations.

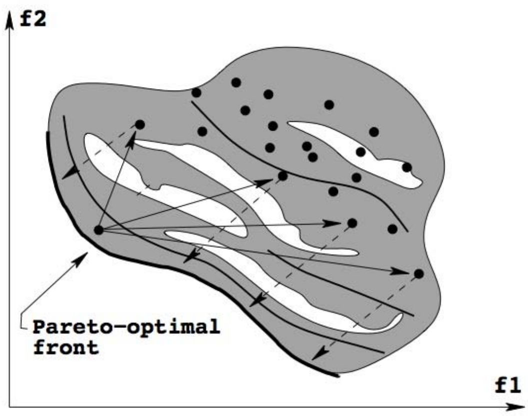

Figure 4. Evolutionary multi-objective optimization algorithm constitutes a parallel search.

Consider Figure 3, in which we sketch how multiple independent parametric single-objective optimizations (through a posteriori MCDM method) may find different Pareto-optimal solutions.

It is worth highlighting here that the Pareto-optimal front corresponds to global optimal solutions of several problems each formed with a different scalarization of objectives. During the course of an optimization task, algorithms must overcome a number of difficulties, such as infeasible regions, local optimal solutions, flat or non-improving regions of objective landscapes, isolation of optimum, etc., to finally converge to the global optimal solution. Moreover, due to practical limitations, an optimization task must also be completed in a reasonable computational time. All these difficulties in a problem require that an optimization algorithm strikes a good balance between exploring new search directions and exploiting the extent of search in currently-best search direction. When multiple runs of an algorithm need to be performed independently to find a set of Pareto-optimal solutions, the above balancing act must have to be performed in every single run. Since runs are performed independently from one another, no information about the success or failure of previous runs is utilized to speed up the overall process. In difficult multi-objective optimization problems, such a memory-less, a posteriori method may demand a large overall computational overhead to find a set of Pareto-optimal solutions (Shukla and Deb, 2007). Moreover, despite the issue of global convergence, independent runs may not guarantee achieving a good distribution among obtained points by an easy variation of scalarization parameters.

EMO, as mentioned earlier, constitutes an inherent parallel search. As explained in Figure 4, when a particular population member overcomes certain difficulties and makes a progress towards the Pareto-optimal front, its variable values and their combination must reflect this fact. When a recombination takes place between this solution and another population member, such valuable information of variable value combinations gets shared through variable exchanges and blending, thereby making the overall task of finding multiple trade-off solutions a parallelly processed task. We shall demonstrate this aspect of parallel processing of population members through a simulation study in Section 2.

3. A Brief Time-line of the Development of EMO Methodologies

During the early years, EA researchers have realized the need of solving multi-objective optimization problems in practice and mainly resorted to using weighted-sum approaches to convert multiple objectives into a single goal (Fogel et al., 1966; Rosenberg, 1967).

However, the first implementation of a real multi-objective evolutionary algorithm (vector-evaluated GA or VEGA) was suggested by David Schaffer in the year 1984 (Schaffer, 1984). Schaffer modified the simple three-operator genetic algorithm (Holland, 1975; De Jong 2006) (with selection, crossover, and mutation) by performing independent selection cycles according to each objective. The selection method is repeated for each individual objective to fill up a portion of the mating pool. Then the entire population is thoroughly shuffled to apply crossover and mutation operators. This is performed to achieve the mating of individuals of different subpopulation groups. The

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Biographical Sketch

Kalyanmoy Deb is Koenig Endowed Chair Professor of Electrical and Computer Engineering at Michigan State University, East Lansing, USA. Prior to this position, he was Gurmukh and Veena Mehta Chair Professor at Indian Institute of Technology Kanpur, India. Prof. Deb received his Bachelor's degree in Mechanical Engineering from Indian Institute of Technology Kharagpur in India in 1985. He received his master's and PhD degrees in Engineering Mechanics from University of Alabama, Tuscaloosa, USA in 1989 and 1991, respectively.

Prof. Deb's main research interests are in evolutionary optimization algorithms and their application in optimization and machine learning. He is largely known for his seminal research in developing and applying Evolutionary Multi-Criterion Optimization (EMO). He was awarded the prestigious Infosys prize, CajAstur Mamdani Prize in Soft Computing, Edgeworth-Pareto award from International Society on MCDM, Shanti Swarup Bhatnagar Prize in Engineering Sciences, Thomson Citation Laureate Award from Thompson Reuters for having the highest number of citations in Computer Science in India. His 2002 IEEE-TEC NSGA-II paper is judged as the Most Highly Cited paper and a Current Classic by Thomson Reuters having more than 11,000 Google Scholar citations so far. He is a fellow of IEEE, ASME, Indian National Science Academy (INSA), Indian National Academy of Engineering (INAE), Indian Academy of Sciences (IASc), and International Society of Genetic and Evolutionary Computation (ISGEC). He has received Friedrich Wilhelm Bessel Research award from Humboldt Foundation in 2003. He has written two text books on optimization and more than 375 international journal and conference research papers. He is in the editorial board on 18 major international journals. More information about his research can be found from http://www.egr.msu.edu/~kdeb.